

## 1 Longest Prefix

Fill in the `longestPrefixOf(String word)` method below such that it returns the longest prefix of `word` that is also a prefix of a key in the trie.

For example, if a `TrieSet t` contains keys `{"cryst", "tries", "cr"}`, then `t.longestPrefixOf("crystal")` returns `"cryst"` and `t.longestPrefixOf("crys")` returns `"crys"`.

The code uses the `StringBuilder` class to build strings character-by-character. To add a character to the end of the `StringBuilder`, use the `append(char c)` method. Once all characters have been appended, the resulting `String` is returned by the `toString()` method.

```
StringBuilder sb = new StringBuilder();
sb.append('a');
sb.append('b');
System.out.println(sb.toString()); // "ab"
```

```
public class TrieSet {
    private Node root;
    private class Node {
        boolean isKey;
        Map<Character, Node> map;
        private Node() {
            isKey = false;
            map = new HashMap<>();
        }
    }
}

public String longestPrefixOf(String word) {
    int n = word.length();
    StringBuilder prefix = new StringBuilder();
    Node curr = _____;
    for (_____ ) {
        _____
        _____
        _____
        _____
        _____
    }
    return _____
}
}
```

## 2    A Tree Takes on Graphs

Your friend at Stanford has come to you for help on their homework! For each of the following statements, determine whether they are true or false; if false, provide counterexamples.

(a) "A graph with edges that all have the same weight will always have multiple MSTs."

(b) "No matter what heuristic you use, A\* search will always find the correct shortest path."

(c) "If you add a constant factor to each edge in a graph, Dijkstra's algorithm will return the same shortest paths tree."

### 3 Class Enrollment

You're planning your CS classes for the upcoming semesters, but it's hard to keep track of all the prerequisites! Let's figure out a valid ordering of the classes you're interested in. A valid ordering is an ordering of classes such that every prerequisite of a class is taken before the class itself. Assume we're taking one CS class per semester.

- (a) The list of prerequisites for each course is given below (not necessarily accurate to actual courses!). Draw a graph to represent our scenario.

- CS 61A: None
- CS 61B: CS 61A
- CS 61C: CS 61B
- CS 70: None
- CS 170: CS 61B, CS 70
- CS 161: CS 61C, CS 70

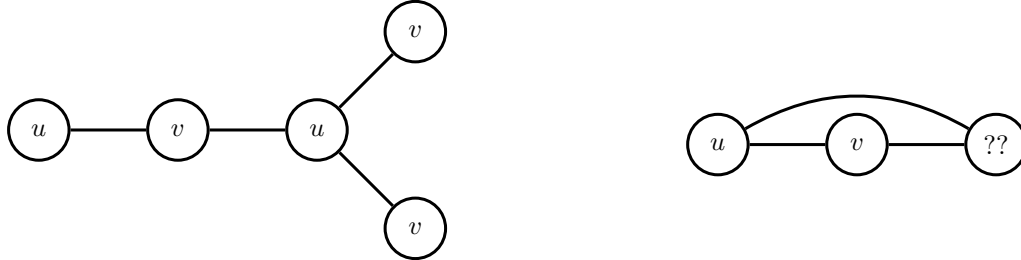
- (b) Suppose we added a new prerequisite where the student must take CS 161 before CS 170 and CS 170 before CS 61C. Is there still a valid ordering of classes such that no prerequisites are broken? If no, explain.

- (c) With the original graph, perform a topological sort to find a valid ordering of the 6 classes. Break ties by going to the lower course number first.

## 4 Graph Algorithm Design

- (a) An undirected graph is said to be bipartite if all of its vertices can be divided into two disjoint sets  $U$  and  $V$  such that every edge connects an item in  $U$  to an item in  $V$ . For example below, the graph on the left is bipartite, whereas on the graph on the right is not. Provide an algorithm which determines whether or not a graph is bipartite. What is the runtime of your algorithm?

*Hint:* Can you modify an algorithm we already know (ie. graph traversal)?



- (b) Consider the following implementation of DFS, which contains a crucial error:

```

create the fringe, which is an empty Stack
push the start vertex onto the fringe and mark it
while the fringe is not empty:
    pop a vertex off the fringe and visit it
    for each neighbor of the vertex:
        if neighbor not marked:
            push neighbor onto the fringe
            mark neighbor
  
```

First, identify the bug in this implementation. Then, give an example of a graph where this algorithm may not traverse in DFS order.

*Hint:* When should we be marking vertices?

- (c) *Extra:* Provide an algorithm that finds the shortest cycle (in terms of the number of edges used) in a directed graph in  $O(EV)$  time and  $O(E)$  space, assuming  $E > V$ .